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Design of Adaptive Structures for Improved Load Capacity

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Application of adaptive structural interfaces (members with actively controlled local properties) as energy dissipaters is discussed. It is demonstrated that the optimal strategy of adaptation of properly located devices allows accumulation of strain energy (as a pretensioned arc) in the first instant of the overloading and then release of this energy in a controlled way, which leads to a significant increase in the overall energy dissipation. The problem of optimal location for dissipaters and the corresponding (controlled) yield stress is formulated and decomposed into two subproblems: first, determining the optimal stress redistribution, and second, selecting the best location for dissipaters and the deformation component due to plastic-like distortions, generated in adaptive members. The general algorithm for the solution of the problem is illustrated by a numerical example of a truss-beam structure.

Introduction

THE problem of safe, self-adaptive structures with improved extreme load capacity is discussed. Supporting structures for landing gear in planes and space vehicles as well as twin hulls in fast ferries are examples of truss structures exposed accidentally to extreme loading due to abnormal landing conditions or collision with an obstacle, respectively. Another example is a light truss space structure under unpredictable collision with an object in the space. Protection against total collapse (provoked by local fracture or concentrated plastic hinges) is the crucial task in these cases. Therefore, the main point in safety design is incorporation of the largest portion of the structure into the plastic-like energy dissipation process in the discussed extreme situations. This concept is well known and explored but mostly as the so-called passive design where highly redundant micromechanical structures (such as honeycomb packages) are applied. The overall effect of these features is comparable to application of structural members with elastoplastic properties, built into the structure, designed for the most expected extreme load. In our proposition, the effect of active adaptation of the yield limits in several chosen members to the detected overloading will be applied, leading to an important increase in energy dissipation and the corresponding load capacity.

The goal of this paper is to present a new formulation of the problem (based on the adaptive structure concept) and the methodology of its solution, enabling determination of the limits of energy dissipation in adaptive structures, as well as compromise results for real applications.

As the first step to adaptive structures design, the static problem of optimal location of adaptive members, as well as optimal distribution of stress limits (for an arbitrary external load) to maximize the energy dissipation, will be discussed. Development of this concept for the dynamic problems of impact absorption will be discussed in a separate paper.

Precomputed numerical results (controlled yield limits in chosen members) can be treated as the so-called desired control, applicable to real-time control strategies of an adaptive structure. These modifications of the yield limits and their adaptation to the detected (in real time) extreme loads will be possible by application of special devices (so-called adaptive energy dissipaters). One possible realization, applied in the project dedicated to experimental verification of the proposed concept, is shown in Fig. 1a, where a hydraulic actuator with controlled valves, opening the flow of fluid, is used. Assume that the characteristic of an element (Fig. 1b) defines desired maximum upper and lower limit values σ^u and σ^l beyond which a plastic behavior occurs. The dissipater device, controlled (by opening, if necessary, valve A_1 or A_2) to keep pressures in the cylinder (Fig. 1a) $p_1 \leq \sigma^l$ and $p_2 \leq \sigma^u$, provokes plastic-like overall behavior of the member. In this way, the original structure equipped with properly located dissipaters can behave like an ideal elastoplastic structure with controllable limit stress level. Alternatively, the dissipater device can be controlled (with modified rules for opening of the valves) to simulate the hardening effect.

The testing structure, shown in Fig. 2, will be equipped with 1) a sensor system detecting and identifying the extreme loading, 2) the energy dissipating devices capable of provoking the generation of local distortions in a controlled way, and 3) a controller realizing in real time the predesigned control strategy of yield stress modifications in dissipaters. The overall effect of such a system of dissipaters is an instantaneous failure (plastic-like adaptation) of a maximal number of structural members as a response particularly adapted to the observed overloading.

The considerations will be constrained to the truss structural mode. However, the concept can be generalized for frame structures as well. At the beginning, we assume the ideal situation when all members can be adaptive (equipped with dissipative devices). Afterwards, a compromise solution, allowing us to apply an arbitrary number of dissipaters, will be discussed. Possible applications of the proposed concept to the dynamic problems of impact absorption, e.g., crashworthiness of rolling stock and adaptive car bumpers, could, eventually, require faster response of actuators because time delay will be the most restricting limitation for the concept feasibility.

Virtual Distortion Method Concept

The design of the system is characterized by the limit stresses that will trigger plastic behavior in the dissipaters. However, for a specified set of such limit stresses, the calculation of the energy absorbed by the system requires a nonlinear analysis that traces the load displacement history.

If, instead, the design variables are selected to be the amounts of plastic deformation in each dissipater, the analysis is greatly

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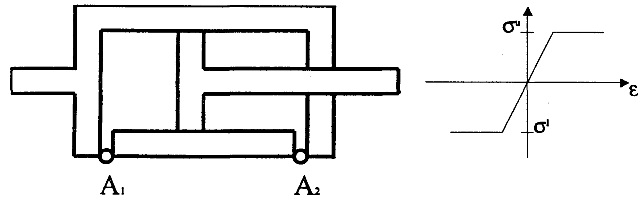


a) b)

Fig. 1 Adaptive energy dissipater.

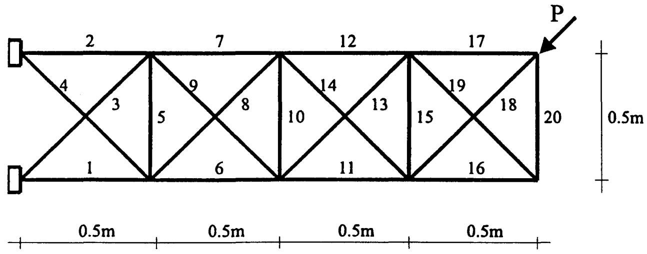


Fig. 2 Truss structure example.

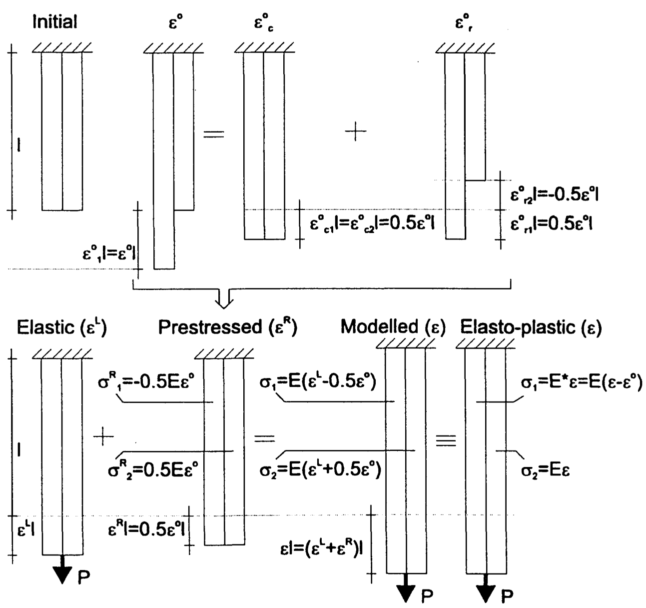


Fig. 3 VDM concept scheme.

simplified. Because the plastic deformations are accompanied by no change in stress, they have the same effect as initial strains (for example, due to heating the member) of the same magnitude. The analysis of a structure with a given set of initial strains is a linear problem that is easy to solve and obtain the corresponding stresses in each member. For members with given plastic deformations, the stresses found from the linear analysis are the limit stresses required for these plastic deformations to occur.

The initial strains used to simulate the plastic deformation are called virtual distortions in the framework of the virtual distortion method,[1] which also uses initial strains to simulate the effect of changes in member cross section.

A simple two-bar truss has been chosen for demonstration of the applied nomenclature.[2] Assume that the left-hand element of the initial structure, shown in Fig. 3, has been subjected to a variation of its configuration, e.g., due to a plastic deformation.

Let us call the corresponding initial strain (permanent plastic deformation) of the left-hand element (in isolation), expressed by ε^0, a virtual distortion. This initially deformed member built into the structure provokes a self-equilibrated state of residual stresses σ^R and strains ε^R (see the prestressed structure in Fig. 3).

Then, let us apply external force-type load P to the structure shown in Fig. 3. It generates the deformation denoted by ε^L in the linearly elastic structure. Superposing these two states of the prestressed and the elastic structure, we get, as a result, a modeled structure (with a combination of linearly elastic responses to initial strains and external load). It is postulated now that (as shown in Fig. 3) the modeled structure is identical in terms of final deformations ε and internal forces with the elastoplastic structure (with elastoplastic response to the load).

Generalizing considerations for any truss structure and looking for the transformation between virtual distortions and the corresponding plastic-like yield stresses, the influence matrix D_{ij}, describing the stresses caused in the truss member i by the unit virtual distortion $\varepsilon_j^0 = 1$ generated in the member j, has to be defined. This matrix stores information about the entire structure properties (including boundary conditions in calculation of structural response). It can be demonstrated (making use of Betti's reciprocal theorem) that by multiplying each row of the matrix $D_{ij} \times A_i l_i$ (where A_i and l_i denote cross-sectional area and the length of the member i, respectively) it becomes symmetric. The rank of the $n \times n$, symmetric matrix $A_i l_i D_{ij}$, where n denotes the number of all members, is equal to the redundancy of the structure. It means that only $k = \mathrm{rank}(A_i l_i D_{ij})$ linearly independent states of initial stresses σ_i^R, where

$$\sigma_i^R = \Sigma_j D_{ij} \varepsilon_j^0 \tag{1}$$

can be introduced into the structure due to generation of an arbitrary state ε_j^0. Looking now for plastic deformations, accompanying adaptation in the plastified elements with stress limits σ_i^u, the following set of linear equations has to be solved with respect to ε_j^0:

$$\sigma_i = \sigma_i^R + \sigma_i^L = \Sigma_j D_{ij} \varepsilon_j^0 + \sigma_i^L \tag{1a}$$

where i and j correspond to plastified members and σ_i^L denotes linear structural response to external load. With this formula the transformation of design variables can be performed.

It will be demonstrated that the strategy of energy dissipation based on the two-stage concept, 1) maximization of strain energy accumulated in the structure due to stress redistribution (up to the critical state in the elastoplastic structural behavior) and 2) maximization of energy dissipation through controlled, kinematically admissible movements performed at previously generated, unchanged stress state, is very effective. Therefore the following decomposition of the virtual distortion state will be useful in further analysis. An arbitrary state of distortions can be uniquely decomposed into two components (cf. Ref. 1, p. 8): a compatible part of deformations $\varepsilon_{ci}^0 \equiv \varepsilon_i^R = \Sigma_j (D_{ij} + \delta_{ij}) \varepsilon_j^0 / E$ (where δ_{ij} is Kronecker's symbol) and the part $\varepsilon_{ri}^0 \equiv -\sigma_i^R / E$ corresponding to the produced stress state:

$$\varepsilon_i^0 = \varepsilon_{ci}^0 + \varepsilon_{ri}^0 = \varepsilon_i^R - \sigma_i^R / E \tag{2}$$

The two distortion components are orthogonal in the sense of the scalar product

$$\Sigma_i A_i l_i \varepsilon_{ci}^0 \varepsilon_{ri}^0 = \Sigma_i A_i l_i \sigma_i^R \varepsilon_i^R / E = 0 \tag{3}$$

The component ε_c^0 is responsible for the compatible, stress-free deformation of the structure, e.g., caused by homogeneous heating of both elements of the truss in Fig. 3, whereas the component ε_r^0 causes the self-equilibrated, strain-free stress state in the structure, e.g., caused by heating of the left element with simultaneous cooling of the right one; see Fig. 3.

Let λ and ε_i^0 be an eigenvalue and a normalized eigenvector of the matrix $A_i l_i D_{ij}$, respectively, which leads to the following relation: $\Sigma_i \Sigma_j \varepsilon_i^0 A_i l_i D_{ij} \varepsilon_j^0 = \lambda$, where $\Sigma_j D_{ij} \varepsilon_j^0 = \sigma_i^R$ describes the self-equilibrated stresses induced by ε_i^0. Therefore, taking into account the decomposition (2), the discussed eigenvalue can be expressed as $\lambda = -\Sigma_i A_i l_i \sigma_i^R \sigma_i^R / E$ and is consequently negative.

Problem Formulation

Let us now formulate the objective function describing dissipation of energy caused in the structure by an arbitrary external load

$$f = \Sigma_i A_i l_i \sigma_i \varepsilon_i^0 \tag{4}$$

where

$$\sigma_i = \sigma_i^R + \sigma_i^L = \Sigma_j D_{ij}\varepsilon_j^0 + \sigma_i^L \quad (5)$$

and vector σ_i^L denotes the stress distribution due to the external load applied. The vector ε_i^0 describes dissipative, plastic-like distortions (to be generated by means of the applied dissipative devices) fulfilling the following conditions, for all $i = 1, 2, \ldots, n$:

$$\varepsilon_i^0 \sigma_i \geq 0 \quad (6)$$

which means that the prestress effect requiring an external source of energy is excluded from the consideration. We assume the elastoplastic structural behavior with the stress limit σ^u and with the following yield conditions:

$$|\sigma_i| \leq \sigma^u \quad (7)$$

Let the stress vector σ_i^L, caused by an arbitrary external force system, have all nonvanishing components. In the opposite case, the totally unloaded members can be excluded from the consideration. It follows from the virtual work principle (cf. also Ref. 1) that the stress field $\sigma_i^L = E\varepsilon_i^L$, where E denotes Young's modulus, corresponding linearly to the compatible field of deformations (due to a force type of load), is orthogonal to all fields of self-equilibrated stresses σ_j^R caused by the unit states of distortions $\varepsilon_j^0 = 1$ [cf. Eq. (1)], for all $j = 1, 2, \ldots, n$,

$$\Sigma_i A_i l_i \sigma_i^R \sigma_i^L = 0 \quad (8)$$

The objective function (4) can be expressed, in view of Eqs. (2), (3), and (8), in the following form:

$$f = f_r + f_c = \Sigma_i \Sigma_j A_i l_i D_{ij} \varepsilon_{rj}^0 \varepsilon_{ri}^0 + \Sigma_i A_i l_i \sigma_i^L \varepsilon_{ci}^0 \quad (9)$$

where the first, quadratic part corresponds to the k-dimensional space of self-equilibrated stresses caused by the component ε_{ri}^0 of virtual distortions, whereas the second, linear part corresponds to the n-k-dimensional space of compatible deformations caused by the component ε_{ci}^0. Also, substituting Eq. (5) into Eq. (7), we can show that the latter condition constrains only the component ε_{ri}^0 of the solution. The matrix $A_i l_i D_{ij}$ is nonpositive definite, which means that all k nonvanishing eigenvalues are negative. The fact follows from the general analysis of initial stresses caused by virtual distortions.

Let us now consider the problem of maximal dissipation of energy f induced in the structure by an arbitrary external load, subject to constraints (6), (7), and

$$|\varepsilon_i^0| \leq \varepsilon^u \quad (10)$$

where ε^u denotes the limit for admissible virtual distortions (due to the piston movement limitation in a dissipater), which leads to some solution

$$\varepsilon_i^* = \varepsilon_{ci}^* + \varepsilon_{ri}^* \quad (11)$$

The preceding solution (of the quadratic programming problem, step 1) corresponds to virtual distortions generated in all members of the structure and determines the limit on maximal energy dissipation. The second question emerges, however: how to maximize energy dissipation with a limited number of dissipaters, which leads to a much more difficult optimization problem, including design of the best location for the dissipative devices.

In our proposition we will demonstrate that instead of dealing with the preceding general problem we can focus our considerations on energy dissipation with the use of a reasonable number of $2k$ (where k is the truss redundancy) adaptive members, which leads (through an inexpensive numerical procedure) to a significant increase of the objective function. The role of the first set of k adaptive members in our strategy will be stress redistribution to maximize stress level in all devices, whereas the role of the second set of k adaptive members will be release of the stored strain energy through k linearly independent, simultaneously generated stress-free modes of pure shape deformations. To this end, let us superpose the solution (11) with another component ε_i^0, geometrically compatible and therefore not affecting the stress distribution. In this way, the overall structure deformation can be significantly reduced with unchanged local stresses σ_i, contributing to the objective function (4) value.

This second problem (step 2) can be formulated as the following linear programming problem:

$$\min f = \min \Sigma_j A_i l_i \sigma_i (\varepsilon_i^* + \varepsilon_i^0) \quad (12)$$

(where $\sigma_i = \Sigma_j D_{ij}\varepsilon_j^* + \sigma_i^L$) subject to Eq. (6), i.e., $(\varepsilon_i^* + \varepsilon_i^0)\,\sigma_i \geq 0$; Eq. (10), i.e., $|\varepsilon_i^0 + \varepsilon_i^*| \leq \varepsilon^u$; and the following constraint:

$$\Sigma_i \Sigma_j A_i l_i D_{ij} \varepsilon_j^* \varepsilon_i^0 = 0 \quad (13)$$

where $\varepsilon_i^* = \varepsilon_{ci}^* + \varepsilon_{ri}^* =$ const is the solution (11) of step 1.

It will be demonstrated in the next section that the side effect of the step 2 solution is determination of the best location for k (where k is redundancy of the structure) dissipaters maximizing the component f_r of the objective function (9). Then, an algorithm for selection of the second set of k adaptive members, e.g., elements with the largest contribution to the objective function, will be proposed. Finally, solving the step 1 problem, but for determined location of $2k$ dissipaters, the corresponding increment of the energy dissipation for this compromise solution can be computed.

Step 1 leads to the solution with the stress redistribution maximizing the first component f_r of the objective function (9) and with the compatible component ε_{ci}^* maximally developed, up to the arbitrarily imposed limits (10), whereas step 2 extracts this unwanted contribution to the final state of dissipative distortions. As $\varepsilon_i^* =$ const and $\varepsilon_i^0 \perp \varepsilon_i^*$, step 2 minimizes the second component f_c of the objective function (9) (measure of the resultant deformation due to $\varepsilon_i^* + \varepsilon_i^0$) subject to the necessary constraints. The constraints (13) (not imposed on the postulated solution in step 1) allow only the compatible component of distortions in step 2.

Further simplification of the k element set selection is based on the fact that imposing constraints (6) in the step 1 problem is not necessary to have the final result (step 1 + step 2) satisfy these conditions. Therefore, the step 1 problem has been modified in the further considerations (step 1a) neglecting conditions (6), which simplifies the problem as all constraints are linear in this case. The original step 1 formulation sometimes causes numerical problems due to singularity of the matrix describing linear constraints (7).

Having the solution of step 1a, the limit for maximal energy dissipation can be evaluated through the following linear programming problem.

Step 2a:

$$\max f = \max \Sigma_j A_i l_i \sigma_i (\varepsilon_i^* + \varepsilon_i^0) \quad (14)$$

(where $\sigma_i = \Sigma_j D_{ij}\varepsilon_j^* + \sigma_i^L$) subject to Eq. (6), i.e., $(\varepsilon_i^* + \varepsilon_i^0)\sigma_i \geq 0$; Eq. (10), i.e., $|\varepsilon_i^0 + \varepsilon_i^*| \leq \varepsilon^u$; and Eq. (13), where $\varepsilon_i^* =$ const is the solution (11) of the modified step 1a optimization problem.

If the solution of step 1a happens to satisfy conditions (6), the result reached through the step 1a + step 2a problem is the same as through the original step 1. We can also expect that if the corresponding violation of constraints (6) is small, then these two results should not differ much.

Numerical Solution

The objective function of the step 1a problem can be expressed as follows:

$$f(\boldsymbol{x}) := \boldsymbol{x}^T(\boldsymbol{A}\boldsymbol{x} + \boldsymbol{b}) \quad (15)$$

where $\boldsymbol{A} := A_i l_i D_{ij} \in R^{n\times n}$, $\boldsymbol{b} := A_i l_i \sigma_i^L \in R^n$, and $\boldsymbol{x} := \varepsilon_i^0 \in R^n$.

The matrix $\boldsymbol{A}$ is real, symmetric, nonpositive definite (which follows from the nonpositive definition of $A_i l_i \boldsymbol{D}$) and $\mathrm{rank}(\boldsymbol{A}) = \mathrm{rank}(A_i l_i \boldsymbol{D}) = k$. Moreover, $\nabla f(\boldsymbol{x}) = 2\boldsymbol{A}\boldsymbol{x} + \boldsymbol{b}$. The spectral decomposition of $\boldsymbol{A}$ takes the form

$$\boldsymbol{A} = \lambda_1 \boldsymbol{v}_1 \boldsymbol{v}_1^T + \lambda_2 \boldsymbol{v}_2 \boldsymbol{v}_2^T + \cdots + \lambda_k \boldsymbol{v}_k \boldsymbol{v}_k^T \quad (16)$$

where k real eigenvalues $\lambda_1 \leq \lambda_2 \leq \cdots \leq \lambda_k \leq 0$ and the corresponding eigenvectors $\boldsymbol{v}_i$ define an orthogonal basis in the k-dimensional subspace of $L_r := \mathrm{span}\{\boldsymbol{v}_1, \boldsymbol{v}_2, \ldots, \boldsymbol{v}_k\}$. Introducing the second subspace $L_c := \{v_{k+1}, v_{k+2}, \ldots, v_n\}$, an arbitrary vector $x \in R^n$ can be expressed as the following sum [cf. Eq. (2)]:

$$x = x_r + x_c \tag{17}$$

where $x_r \in L_r$, $x_c \in L_c$, and $x_r^T x_c = 0$.

Denoting rows of the matrix A by a_i, $i = 1, 2, \ldots, n$, the step 1a problem leads to the following quadratic programming problem:

$$\max x^T A x + b^T x \tag{18a}$$

subject to the conditions

$$\left|a_i^T x + b_i\right| \le \sigma^u \tag{18b}$$

$$|x_i| \le \varepsilon^u \tag{18c}$$

where σ^u and ε^u are positive constants. The set of the feasible solutions is compact. If it is not empty, the continuity of the objective function implies the existence of the optimal solution x^*, which can be decomposed in the following way [cf. Eq. (17)]:

$$x^* = x_r^* + x_c^* \tag{19}$$

The analytical expression for the gradient of the objective function, $\nabla f(x) = 2Ax + b$, can be used in the optimization procedure DTOLMIN (Ref. 3), which is a double precision version of the subroutine TOLMIN (Ref. 4). The underlying method (a variant of the active set method) is described by Powell.[5] This method is particularly suitable if there are any degenerate and nearly degenerate linear constraints (as in our problem).

Since $b \in L_c$, [cf. Eq. (8)] and taking into account the orthogonal decomposition of the solution x^*, the objective function can be expressed as follows [cf. Eq. (9)]:

$$f(x) = \left(x_r^* + x_c^*\right)^T \left[A\left(x_r^* + x_c^*\right) + b\right] = \cdots = \left(x_r^*\right)^T A x_r^* + \left(x_c^*\right)^T b \tag{20}$$

Now, step 2 leads (ignoring the component x_c^* of the preceding solution and taking the component x_r^* as fixed) to the following linear programming problem:

$$\min b^T y + \left(x_r^*\right)^T A x_r^* \tag{21a}$$

where the second component of the objective function is constant, while the unknown vector $y \in R^n$ is orthogonal to L_r [cf. Eq. (13)]:

$$\begin{aligned} y^T v_1 &= 0 \\ y^T v_2 &= 0 \\ &\cdots \\ y^T v_K &= 0 \end{aligned} \tag{21b}$$

and satisfies (together with x_r^*) constraints (6) and (10):

$$\left[\left(x_r^*\right)_i + y_i\right]\left(a_i^T x_r^* + b_i\right) \ge 0 \tag{21c}$$

$$\left|\left(x_r^*\right)_i + y_i\right| \le \varepsilon^u \tag{21d}$$

where $(x_r^*)_i$ and y_i denote ith components of vectors x_r^* and y, respectively. The problem (21), with linearly independent constraints (21b) expressed through the auxiliary unknowns $z = x_r^* + y$ and with the n constraints (21c) converted to $2n$ simple bounds, can be presented in the standard form of the linear programming problem. It is known from the linear programming theory that if the feasible region is not empty and if the solution exists, then in the set of the optimal solution there exists at least one vector (so-called basic feasible solution) with not more than k nonzero components.[6] The problem (21) can be solved by the numerically stable version of the simplex method, based on the QR decomposition and its rank-one updating strategy via Givens rotations.[3]

Let us conclude the preceding considerations with the following corollary: If there exists a solution for the problem of maximum energy dissipation through the stress redistribution (step 1 + step 2), it can be realized with dissipative distortions generated in k elements only, where k is the redundancy of the structure.

It is important that the optimal location of these k adaptive members of the structure can be determined as a side effect of the simplex method, without the expensive integer programming procedure. The corresponding step 2a of the maximum energy dissipation problem leads to the following programming problem:

$$\max b^T y + \left(x_r^*\right)^T A x_r^* \tag{22}$$

subject to constraints (21b), (21c), and (21d). We can expect a solution with a large number of active constraints (21d) in this case.

Let us demonstrate the concept of energy dissipaters on the simple truss structure example shown in Fig. 2 and loaded statically by the force P. Assuming uniformly distributed material properties $E = 1.0$ MPa, member cross sections $A = 1.60$ cm^2, and the yield stress $\sigma^u = 1.2$ kPa, the critical load $P_c = 0.0907$ N (causing total collapse with plastified members 1 and 2) can be determined. Allowing plastic flow up to the locally measured limits $|\varepsilon^0| \le 0.03$, the capacity of energy dissipation reaches the value $U = 5.760 \times 10^{-3}$ J (case NP_c in Fig. 4) due to the geometrically variable component caused by distortions generated in members nos. 1 and 2.

On the other hand, solving the problem of maximum dissipation of load energy, but mainly due to the stress redistribution (step 1a + step 2, with minimized influence of overall deformation) for the load $P_0 = 0.0792$ N (close to the maximal load with pure elastic response of the structure), the value $U = 0.014 \times 10^{-3}$ J, due to distortions generated in members 3, 9, 13, and 19 of the following values, $\varepsilon_3^0 = -0.00003$, $\varepsilon_9^0 = 0.00025$, $\varepsilon_{13}^0 = -0.00042$, and $\varepsilon_{19}^0 = 0.00025$, can be reached (case SP_0 in Fig. 4). Resultant stresses (5) take the corresponding values (Table 1).

The preceding result can grow significantly, up to $U = 16.798 \times 10^{-3}$ J, if the contribution of the component ε_c^0 is not restricted (step 1a + step 2a − case AP_0 in Fig. 4). In all members, except for no. 10 (vanishing adaptive distortions) and nos. 5, 8, 13, and 19 (with adaptive distortions $\varepsilon_5^0 = 0.0001$, $\varepsilon_8^0 = -0.0297$, $\varepsilon_{13}^0 = -0.0154$,

Table 1 Stress distribution for the case SP_0

$\sigma_1 = -1200.0$ Pa	$\sigma_6 = -781.3$ Pa	$\sigma_{11} = -396.7$ Pa	$\sigma_{16} = -61.8$ Pa
$\sigma_2 = 897.1$ Pa	$\sigma_7 = 616.7$ Pa	$\sigma_{12} = 102.3$ Pa	$\sigma_{17} = -61.8$ Pa
$\sigma_3 = -280.1$ Pa	$\sigma_8 = -377.9$ Pa	$\sigma_{13} = -144.7$ Pa	$\sigma_{18} = -406.9$ Pa
$\sigma_4 = 214.2$ Pa	$\sigma_9 = 116.4$ Pa	$\sigma_{14} = 349.6$ Pa	$\sigma_{19} = 87.4$ Pa
$\sigma_5 = 115.8$ Pa	$\sigma_{10} = 20.0$ Pa	$\sigma_{15} = 40.3$ Pa	$\sigma_{20} = -61.8$ Pa

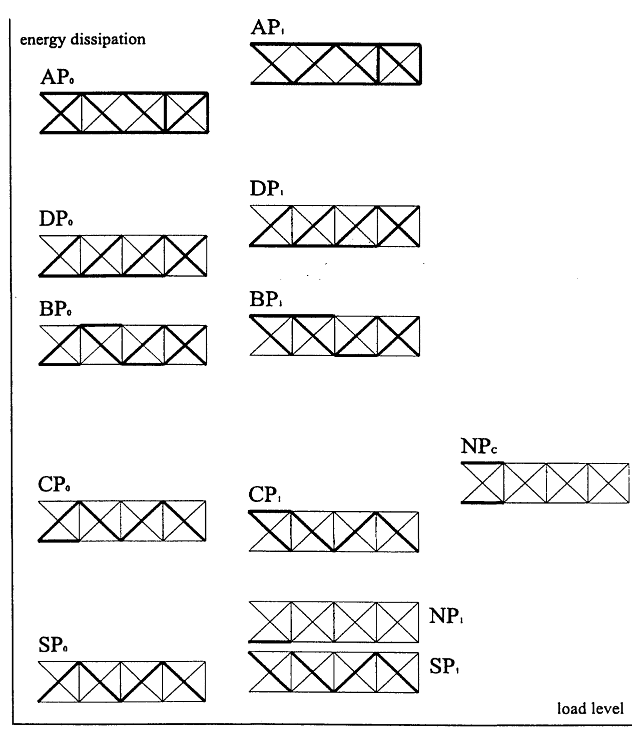


Fig. 4 Location of dissipaters in adaptive structures.

and $\varepsilon_{19}^0 = 0.0247$), the constraints $|\varepsilon^0| \leq 0.03$ are active in this case. The corresponding resultant stresses are the same as in the preceding case SP_0.

If we decide, however, to reduce the number of dissipating elements to five (members 1, 3, 9, 13, and 19), that is, just one more than in the case SP_0 , we achieve the value $U = 3.373 \times 10^{-3}$ J (case CP_0), which is a very substantial growth of energy dissipation in comparison with the case SP_0 (mainly due to the boundary distortion $\varepsilon_1^0 = -0.03$ generated in member 1). Looking farther for a compromise solution (not too many dissipating elements reconciled with a significant value of dissipated energy), we can increase the value up to $U = 8.400 \times 10^{-3}$ J (case BP_0) with eight dissipaters located in members 1, 3, 7, 9, 11, 13, 18, and 19.

Now let us consider the load $P_1 = 1.14 \times P_0 = 0.0903$ N (close to the critical one $P_c = 0.0907$ N), which exploits the structure strength to its maximum but does not cause total collapse. The corresponding solution of the step 1a + step 2 problem leads to the energy dissipation $U = 0.018 \times 10^{-3}$ J due to distortions, generated in members 4, 9, 13, and 19, of the values $\varepsilon_4^0 = 0.00070$, $\varepsilon_9^0 = 0.00028$, $\varepsilon_{13}^0 = -0.00048$, and $\varepsilon_{19}^0 = 0.00028$ (case SP_1 in Fig. 4). Resultant stresses (5) take the corresponding values (Table 2).

The preceding result can grow significantly, up to $U = 19.155 \times 10^{-3}$ J, if contribution of the component ε_c^0 is not restricted (step 1a + step 2a – case AP_1 in Fig. 4). In all members, except for nos. 5 and 10 (vanishing adaptive distortions) and nos. 3, 8, 13, and 19 (with adaptive distortions $\varepsilon_3^0 = -0.0293$, $\varepsilon_8^0 = -0.0297$, $\varepsilon_{13}^0 = -0.0155$, and $\varepsilon_{19}^0 = 0.0282$), the constraints $|\varepsilon^0| \leq 0.03$ are active in this case. The corresponding resultant stresses are the same as in the preceding case SP_1, and the resultant deformations are shown in Fig. 5.

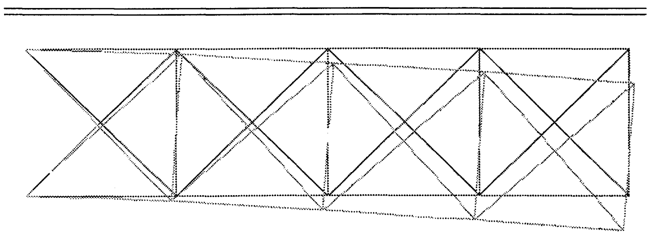
Proceeding in the same way as for load P_0, we obtain the value $U = 2.898 \times 10^{-3}$ J (case CP_1) if five dissipaters in members 2, 4, 9, 13, and 19 are allowed and the value $U = 8.640 \times 10^{-3}$ J (case BP_1) if eight dissipaters in members 2, 4, 7, 9, 11, 13, 18, and 19 are allowed, respectively.

The natural elastoplastic response of the structure without dissipaters for the load P_1 (the case NP_1 in Fig. 4) leads to the energy dissipation $U = Al\sigma^u\varepsilon_1^0 = 1.60\,\text{cm}^2 \times 50\ \text{cm} \times (-1.2)\,\text{kPa} \times (-0.00133) = 0.128 \times 10^{-3}$ J due to the plastic distortion $\varepsilon_1^0 = -0.00133$ generated in member 1.

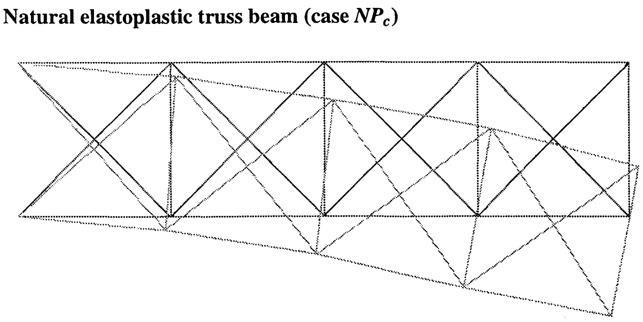
It is interesting to observe that the first plastification of member 1 implies a change of topology in the solution of the step 1a + step 2 problem (cf. cases SP_0 and SP_1).

Table 2 Stress distribution for the case SP_1

$\sigma_1 = -1200.0$ Pa	$\sigma_6 = -910.9$ Pa	$\sigma_{11} = -680.1$ Pa	$\sigma_{16} = -70.8$ Pa
$\sigma_2 = 1196.7$ Pa	$\sigma_7 = 686.9$ Pa	$\sigma_{12} = 118.8$ Pa	$\sigma_{17} = -70.8$ Pa
$\sigma_3 = -362.4$ Pa	$\sigma_8 = -406.3$ Pa	$\sigma_{13} = -168.0$ Pa	$\sigma_{18} = -464.8$ Pa
$\sigma_4 = 2.3$ Pa	$\sigma_9 = 158.3$ Pa	$\sigma_{14} = 396.9$ Pa	$\sigma_{19} = 100.1$ Pa
$\sigma_5 = 285.8$ Pa	$\sigma_{10} = 6.8$ Pa	$\sigma_{15} = 48.0$ Pa	$\sigma_{20} = -70.8$ Pa



Natural elastoplastic truss beam (case NP_c)



Adaptive truss beam (case AP_1)

Fig. 5 Structural deformation.

To conclude the example, let us emphasize that the constraints (6) of the original step 1 problem are violated here only in two vertical members, nos. 5 and 10, which do not contribute much to the objective function (4) value. This fact indicates that, arriving at our solution of the step 1a + step 2a, we are very close to the desired optimum (direct solution of the step 1). Therefore, if we dispose of members 5 and 10, all constraints (6) become inactive, which means that the combined solution of step 1a + step 2a is exactly the same as the direct solution of step 1.

Compromise Solution

The presented example of the ideal structure demonstrates a significant increase in energy dissipation capability in an adaptive structure. Especially for lower subcritical loads, the advantage of the adaptive concept is significant, which can be very useful in applications to impact absorption (in a dynamic model), where energy dissipation has to take place as quickly as possible. It can be observed that location of active elements responsible for the stress redistribution is spread through all four sections (with 1 deg of redundancy each), and the main contribution to the objective function is brought by the elements from the underloaded sections. On the contrary, the contribution of active elements (causing geometrically variable deformations) to the objective function is mainly due to the elements located in the overloaded area. The contribution of the second group of elements is much larger; however, it provokes an undesirable increase in the overall structural deformation and has to be controlled. It is important to combine both phases—the strain energy accumulation due to the stress redistribution and then the strain energy release due to the plastic-like hinge generation—to get a significant energy dissipation.

It has been shown that the optimal location of dissipaters can depend on the load intensity (see Fig. 4, cases SP_0 and SP_1). Also, the main contribution to the objective function comes from spread active elements causing geometrically variable deformations. In real applications, however, the limitation of the number of dissipaters has to be taken into account also. It seems reasonable to assume k actuators, responsible for the stress redistribution, and another k, responsible for geometrically variable modes of virtual distortions. Then the preceding solution of steps 1a and 2 gives as a side effect the optimal location of k actuators, designed to control the stress redistribution (with contribution to the overall deformation kept as small as possible), for the a priori defined load. If this load is unknown (as in the normal case of active control), then the following strategy in design of adaptive structures is justified: 1) determination of the optimal distribution of k active elements capable of controlling an arbitrary state of stresses (a set B of combinations is expected); 2) selection of the best configuration (from the set B) of k active elements, giving the largest contribution to the objective function for several expected load distributions; 3) selection of k additional active elements, responsible for progressive deformation, e.g., maximally loaded (in many load states) members from each section can be chosen; and 4) the solution of step 1a + step 2a problem for the $2k$ active elements determined in the preceding points 2 and 3.

The method of solving point 1, based on the concept of determination of the four-element combination (in the case of the truss structure shown in Fig. 2) giving the best approximation of the quadratic form (15) defined on the subspace spanned by the eigenvectors of the matrix $\boldsymbol{A}$, is presented in Ref. 7. Following this strategy, the location for active elements 3, 8, 13, and 18 controlling stress redistribution (point 2) has been chosen from the set B determined in point 1, as maximally contributing to the objective function for the given load (Fig. 4). Then, the other four elements, nos. 1, 6, 11, and 19, maximally loaded in each section (point 3), have been detected. Finally, solving the general optimization problem (point 4) for the set of adaptive elements 1, 3, 6, 8, 11, 13, 18, and 19, the energy dissipation $U = 9.226 \times 10^{-3}$ J (Fig. 4, case DP_0) and $U = 10.497 \times 10^{-3}$ J (Fig. 4, case DP_1) can be computed. In this way the compromise solution with a limited number of actuators, but significantly contributing to the objective function, can be found. Note that the compromise solution determined here (Fig. 4, cases DP_0 and DP_1) is still slightly better than the solution (Fig. 4, cases BP_0 and BP_1) determined on the basis of the step 1a + step 2a problem. However, the numerical cost of selecting the best combinations of B can cause some problems for large structures.

Conclusions

It has been demonstrated that the capability of extreme load energy dissipation for a structure equipped with adaptive interfaces (controlled dissipaters) can be significantly increased (over three times in the preceding example). It is important that active control deal only with release of valves and therefore require only small external energy input. The efficiency of the discussed concept, however, strongly depends on the proper location of dissipaters as well as on the applied strategy of release of local plastic effects. In this way, the two-step strategy, consisting of 1) accumulation of the strain energy and then 2) its controlled release to enable large deformations, looks very promising.

The comparison of adaptive truss-beam behavior with that of the natural one (Fig. 5) shows that the overall effect of adaptive dissipation is similar to generation of four (instead of one in the natural case) plastic hinges in the bending beam model.

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